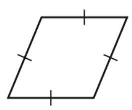
Cornell Notes	Topic/Objective: 7.4 Properties of Special Parallelograms	Name:
		Class/Period: Geometry
		Date:

Essential Question: What are the properties of the diagonals of rectangles, rhombuses, and squares?

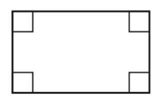
Questions:

Notes:

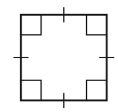
Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.



A **rectangle** is a parallelogram with four right angles.

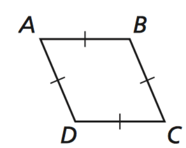


A **square** is a parallelogram with four congruent sides and four right angles.

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

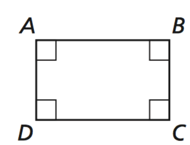
$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.



Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

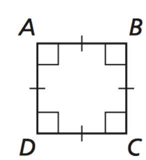
$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

$ABCD$ is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

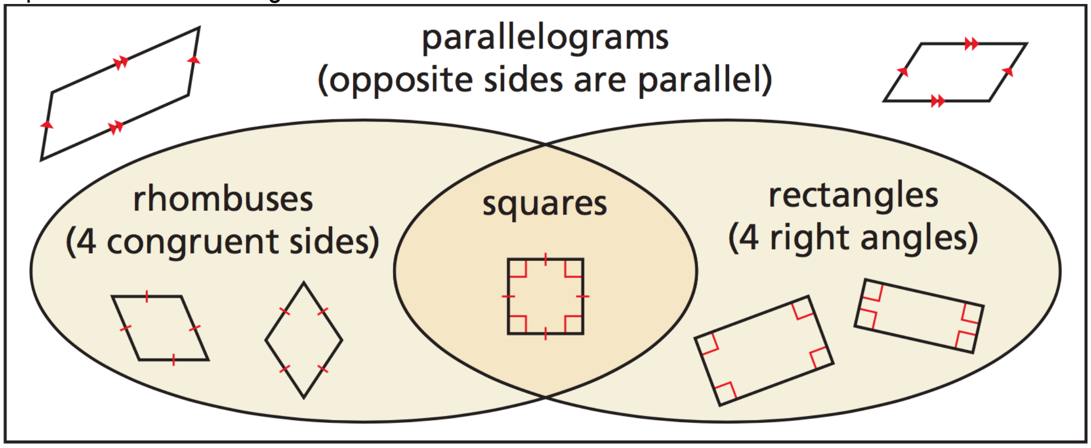


Summary:

Questions:	Notes:

Notes:

The Venn diagram below illustrates some important relationships among parallelograms, rhombuses, rectangles, and squares. For example, you can see that a square is a rhombus because it is a parallelogram with four congruent sides. Because it has four right angles, a square is also a rectangle.



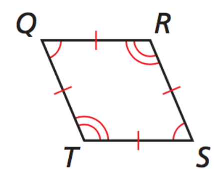
EXAMPLE 1 Using Properties of Special Quadrilaterals

For any rhombus $QRST$, decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

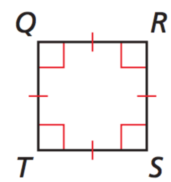
a. $\angle Q \cong \angle S$ **b.** $\angle Q \cong \angle R$

SOLUTION

a. By definition, a rhombus is a parallelogram with four congruent sides. By the Parallelogram Opposite Angles Theorem (Theorem 7.4), opposite angles of a parallelogram are congruent. So, $\angle Q \cong \angle S$. The statement is *always* true.



b. If rhombus $QRST$ is a square, then all four angles are congruent right angles. So, $\angle Q \cong \angle R$ when $QRST$ is a square. Because not all rhombuses are also squares, the statement is *sometimes* true.



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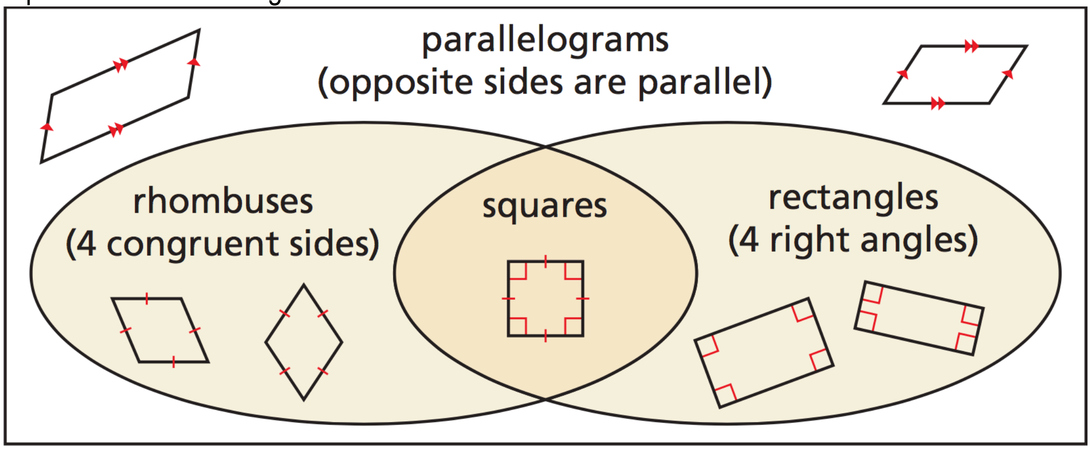

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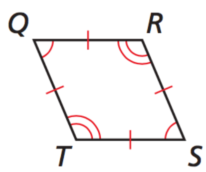

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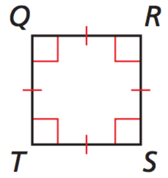

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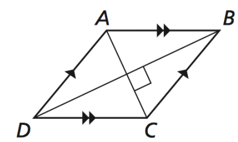
Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

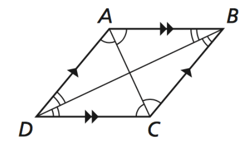
$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.



Theorem 7.12 Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.



EXAMPLE 3 Finding Angle Measures in a Rhombus

Find the measures of the numbered angles in rhombus *ABCD*.

SOLUTION

Use the Rhombus Diagonals Theorem and the Rhombus Opposite Angles Theorem to find the angle measures.

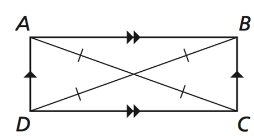
$m\angle 1 = 90°$	The diagonals of a rhombus are perpendicular.
$m\angle 2 = 61°$	Alternate Interior Angles Theorem (Theorem 3.2)
$m\angle 3 = 61°$	Each diagonal of a rhombus bisects a pair of opposite angles, and $m\angle 2 = 61°$.
$m\angle 1 + m\angle 3 + m\angle 4 = 180°$	Triangle Sum Theorem (Theorem 5.1)
$90° + 61° + m\angle 4 = 180°$	Substitute 90° for $m\angle 1$ and 61° for $m\angle 3$.
$m\angle 4 = 29°$	Solve for $m\angle 4$.

So, $m\angle 1 = 90°$, $m\angle 2 = 61°$, $m\angle 3 = 61°$, and $m\angle 4 = 29°$.

Theorem 7.13 Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.



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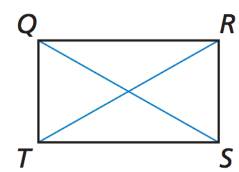
$90° + 61° + m\angle 4 = 180°$	Substitute 90° for $m\angle 1$ and 61° for $m\angle 3$.
$m\angle 4 = 29°$	Solve for $m\angle 4$.

▶ So, $m\angle 1 = 90°$, $m\angle 2 = 61°$, $m\angle 3 = 61°$, and $m\angle 4 = 29°$.

EXAMPLE 5 Finding Diagonal Lengths in a Rectangle

In rectangle *QRST*, $QS = 5x - 31$ and $RT = 2x + 11$. Find the lengths of the diagonals of *QRST*.



SOLUTION

By the Rectangle Diagonals Theorem, the diagonals of a rectangle are congruent. Find x so that $\overline{QS} \cong \overline{RT}$.

$QS = RT$	Set the diagonal lengths equal.
$5x - 31 = 2x + 11$	Substitute $5x - 31$ for QS and $2x + 11$ for RT.
$3x - 31 = 11$	Subtract $2x$ from each side.
$3x = 42$	Add 31 to each side.
$x = 14$	Divide each side by 3.

When $x = 14$, $QS = 5(14) - 31 = 39$ and $RT = 2(14) + 11 = 39$.

▶ Each diagonal has a length of 39 units.

Summary:

Questions:	Notes:

EXAMPLE 6 **Identifying a Parallelogram in the Coordinate Plane**

Decide whether ▱*ABCD* with vertices $A(-2, 6)$, $B(6, 8)$, $C(4, 0)$, and $D(-4, -2)$ is a *rectangle*, a *rhombus*, or a *square*. Give all names that apply.

SOLUTION

1. **Understand the Problem** You know the vertices of ▱*ABCD*. You need to identify the type of parallelogram.
2. **Make a Plan** Begin by graphing the vertices. From the graph, it appears that all four sides are congruent and there are no right angles.

   Check the lengths and slopes of the diagonals of ▱*ABCD*. If the diagonals are congruent, then ▱*ABCD* is a rectangle. If the diagonals are perpendicular, then ▱*ABCD* is a rhombus. If they are both congruent and perpendicular, then ▱*ABCD* is a rectangle, a rhombus, and a square.
3. **Solve the Problem** Use the Distance Formula to find *AC* and *BD*.

$$AC = \sqrt{(-2-4)^2 + (6-0)^2} = \sqrt{72} = 6\sqrt{2}$$

$$BD = \sqrt{[6-(-4)]^2 + [8-(-2)]^2} = \sqrt{200} = 10\sqrt{2}$$

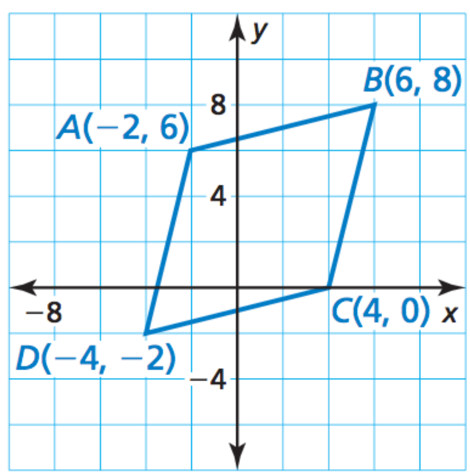
Because $6\sqrt{2} \neq 10\sqrt{2}$, the diagonals are not congruent. So, ▱*ABCD* is not a rectangle. Because it is not a rectangle, it also cannot be a square.

Use the slope formula to find the slopes of the diagonals $\overline{AC}$ and $\overline{BD}$.

$$\text{slope of } \overline{AC} = \frac{6-0}{-2-4} = \frac{6}{-6} = -1 \qquad \text{slope of } \overline{BD} = \frac{8-(-2)}{6-(-4)} = \frac{10}{10} = 1$$

Because the product of the slopes of the diagonals is -1, the diagonals are perpendicular.

▶ So, ▱*ABCD* is a rhombus.



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